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Detection of Weak Lensing in the Fields of Luminous Radiosources

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Abstract. We present a first attempt to reveal the possible existence of large foreground mass condensations directly responsible for the gravitational magnification of four distant luminous radiosources and one optical QSO. The technique uses a weak lensing analysis of the distant galaxies in the field of each source. We find a coherent shear [2] map with a large magnification bias on the line of sight to Q1622+0328. The local shear in the field of the bright radiosources is also often correlated with nearby groups or poor clusters of galaxies. For three of them: PKS0135-247, 3C446 and PKS1508-05, the groups are identified as gravitational deflectors that magnify the radiosources.

This result suggests that a substantial amount of invisible mass is condensed in groups and poor clusters of galaxies. It may explain the origin of a large angular correlation between the distribution of distant radiosources (z >1) and the distribution of low redshift galaxies (z <0.3) (Bartelmann & Schneider 1993).

We discuss the feasability and consequences of a future systematic survey to investigate the problem of magnification bias in the fields of luminous distant objects and to probe the mass distributions of galaxy groups at intermediate redshifts.

Key words: Cosmology: observations – Large-Scale Structure – Gravitational Lensing – Quasars

1. Introduction

It has been suspected for a long time that the large number of extremely bright QSOs or radiosources may arise from magnification by foreground gravitational structures (TWD95 and references therein). Knowledge of the magnification bias function provides constraints on the distribution of mass condensations, the density parameter Ω of the Universe, as well as the luminosity function of QSOs. This has stimulated surveys to search for an excess of galaxies close to the line of sight of the brightest QSOs; however inconclusive results emerged. For small angular separations, the detection is difficult without large coronographs (blooming of the detector, instrumental scattering of light). The results are still controvertial but nevertheless, on small scale (< 5"), an excess has been observed which corresponds to an overdensity of galaxies $q = 1.4 \pm 0.5$ for bright QSOs with M_v <-28 and z>1.5. The result seems to be in agreement with theoretical predictions of magnification bias by individual galaxy lenses (TWD95). Correlations on scales larger than one arcminute gave negative results until **?**fugmann90 announced a correlation between the distribution of 1-Jy distant radiosources (z > 1) and the Lick catalogue of galaxies (z < 0.1). This claim prompted the further statistical analysis of **?**bartelmann93b, who verified Fugmann's result with the sensitive Spearman rank order correlation test. They confirmed the existence of a correlation between the foreground galaxies of both the Lick and IRAS catalogues and radiosources with z>1, on angular scales larger than 10 arcminute (BS93a, BS94)! The test does not give the strength of the correlation but we know that individual lensing galaxies cannot produce correlations on so large a scale (BS93b). A recent photographic plate analysis (BMG95) of the galaxy distribution provides additional observational evidence that the correlation is real.

The lensing hypothesis would more easily explain the correlation if the lensing agents have larger cross sections than galaxies. The most natural candidates are the numer-

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[1] Based on observations at the Canada-France-Hawaii Telescope operated by the french Institut des Sciences de l'Univers (INSU), the Canadian National Research Council (CNRC) and the University of Hawaii (UH), and at the European Southern Observatory (ESO) at La Silla-Chili.

[2] Following the **?**bonnet94 and **?**bonnet95 papers, we will use the term *shear* or *polarisation* as $e = 1 - b/a$.

ous galaxy clumps distributed in the Large Scale Structures of the Universe (hereafter LSS) if a substantial fraction of them have almost the critical surface mass density. In fact, the excess of QSOs and radiosources around the Zwicky, the Abell and the ROSAT clusters reported recently (BSH94, SS95c) already supports the idea that cluster-like structures may play a significant role in magnifying a fraction of bright quasars. If this hypothesis is true these massive, not yet detected deflectors in visible could show up through their weak lensing effects on the background galaxies.

The gravitational weak lensing analysis has recently proved to be a promising technique to map the projected mass around clusters of galaxies (KS93, BMF94, FKSW94, SEF94). Far from the centers of such mass condensations, background galaxies are weakly stretched perpendicular to the gradient of the gravitational field. With the high surface density of background galaxies up to $V = 27.5$ (≈ 43 faint sources per square arcminute with $V > 25$) the local shear (or polarization of the images) can be recovered from the measurement of the image distortion of weakly lensed background galaxies averaged over a sky aperture with typical radius of 30 arcsec. The implicit assumption that the magnification matrix is constant on the scanning aperture is not always valid and this observational limitation will be discussed later

The shear technique was also used with success to detect large unknown deflectors in front of the doubly imaged quasar Q2345+007 (BFK$^+$93). This QSO pair has an abnormally high angular separation, though no strong galaxy lens is visible in its neighbourhood. The shear pattern revealed the presence of a cluster mass offcentered at one arcminute north-east from the double quasar, which contributes to the large angular separation. Further ultra-deep photometric observations in the visible and the near infrared have *a posteriori* confirmed the presence of the cluster centered on the center of the shear pattern and detected a small associated clump of galaxies as well, just on the QSO line of sight. Both lensing agent are at a redshift larger than 0.7 (MDFFB94, FTBG94, PMLB$^+$95). The predicting capability of the weak lensing was quite remarkable since it *a priori* provided a better signature of the presence of a distant cluster than the actual overdensity of galaxies, which in the case of Q2345+007 was almost undectable without a deep "multicolor" analysis.

On a theoretical side, numerical simulations in standard adhesion HDM or CDM models (BS92a) can predict the occurrence of quasar magnification. They have shown that the large magnifications are correlated with the highest amplitudes of the shear, which intuitively means that the largest weak lensing magnifications are in the immediate vicinity of dense mass condensations. For serendipity fields they found from their simulations that at least 6% of background sources should have a shear larger than 5%. However, for a subsample of rather bright radiosources or QSOs the probability should be larger, so that we can reasonably expect quasar fields with a shear pattern above the detection level.

Since we can detect shear as faint as 3% (BMF94), both observational and theoretical arguments convince us to start a survey of the presence of weak shear around several bright radiosources. In practice, mapping the shear requires exceptional subarcsecond seeing (<0.8 arcsec.) and long exposure times, typically 4 hours in V with a four meter class telescope. Observations of a large unbiased selected sample of QSOs will demand several years and before promoting the idea of a large survey we decided to probe a few bright QSO fields where a magnification bias is more likely.

In this paper, we report on a preliminary tests at CFHT and ESO of five sources at $z \approx 1$. The analysis of the shape parameters and the shear is based on the **?**bonnet95 technical paper, with some improvements to measure very weak ellipticities. Due to instrumental difficulties only one, Q1622+238, was observed at CFHT. Nevertheless, we found a strong shear pattern in the immediate vicinity of the quasar quite similar to the shear detected in the QSO lens Q2345+007 (BFK$^+$93). The QSO is magnified by a previously unknown distant cluster of galaxies. The four other QSOs were observed with the imaging camera SUSI at the NTT with a significantly lower instrumental distortion but with a smaller field of view. In this case the limited size of the camera makes the mapping of strong deflector like in Q1622+238 harder. However, with the high image quality of SUSI it is possible to see on the images a clear correlation between the amplitude and direction of the shear and the presence of foreground overdensities of galaxies. Some of them are responsible for a magnification bias of the QSO.

By comparing the preliminary observations at CFHT and ESO we discuss important observational issues, namely the need for a perfect control of image quality and a large field of view. We also show that invisible masses associated with groups and poor clusters of galaxies can be seen through their weak lensing effect with NTT at ESO. These groups of galaxies may explain the origin of a large angular correlation between the distribution of distant radiosources ($z > 1$) and the distribution of low redshift galaxies ($z < 0.3$) The study of the correlation between the local shear and nearby overdensity of foreground galaxies (masses) will be investigated in following papers after new spectrophotometric observations of the lensing groups.

2. Selection and observations of the sources

The double magnification bias hypothesis maximises the probability of a lensing effect for luminous distant sources (BvR91). Therefore whenever possible we try to select sources that are both bright in radio (F> 2Jy, V <18). We also looked at quasars with absorption lines at lower redshift, to know if some intervening matter on the lines of sight is present. The QSOs are chosen at nearly the

object	α_{50}	δ_{50}	m_V	z	flux	Tel./Instr.	exp. time	numb. files	seeing (arcsec.)
PKS0135-247	01 35 17.17	-24 46 11.7	16.98	0.832	1.70	NTT/SUSI	16500	10	0.75
PKS1508-05	15 08 14.94	-05 31 49.0	17.20	1.191	2.43	NTT/SUSI	13500	5	0.76
PKS1741-03	17 41 20.62	-03 48 49.0	19.00	1.057	3.65	NTT/SUSI	23700	13	0.68
3C446	22 23 11.08	-05 12 17.8	18.03	1.404	3.87	NTT/SUSI	19700	9	0.66
Q1622+238	16 22 32.40	+23 52 01.0	18.18	0.927	–	CFHT/FOCAM	18000	10	0.78

Table 1. Observational data for the 5 QSOs fields. The V magnitude stars. The radioflux is the 5009 MHz value from the 1Jy catalogue. The total exposure time corresponds to the coaddition of several individual images with 30-45 minutes exposure time. The seeing is the FWHM of stars on the composite image

mean redshift of the faint background galaxies (z from 0.8 to 1.) used as an optical template to map the shear of foreground deflectors. So far, we have observed 5 QSOs at redshift about 1 with a V magnitude and radio flux in the range from 17 to 19 and 1.7 to 3.85 respectively (Table 1).

Except Q1622+238 (z=0.97) which was suspected to have a faint group of galaxies nearby (HRV91), the 4 other candidates (PKS0135-247, PKS1508-05, PKS1741-03, and 3C446.0) have been only selected from the **?**hewitt87 , and the **?**veronveron85 catalogues, choosing those objects with good visibility during the observing runs. The V magnitude of each QSO was determined with an accuracy better than 0.05 mag. rms from faint **?**landolt92 calibration stars (Table 1).

The observations started simultaneously in June 1994 at the ESO/NTT with SUSI and at CFHT with FOCAM, both with excellent seeing conditions (<0.8") and stable transparency. For the second run at ESO in November 1994, only one of the two nights has good seeing conditions for the observation of PKS0135-247. We used the 1024×1024 TeK and the 2048×2048 LORAL CCDs with 15 micron pixel, which correspond to 0.13"/pixel at the NTT and 0.205"/pixel at CFHT, and typical fields of view of 2' and 7' respectively. In both cases we used a standard shift and add observing technique with 30 to 45 min exposures. The resulting field of view is given in table 2. The total exposure was between 16500 and 23700 seconds in V (Table 1). The focusing was carefully checked between each individual exposure. After prereduction of the data with the IRAF software package, all frames were coadded leading to a composite image with an effective seeing of 0.78" at CFHT and 0.66"-0.78" at NTT (Table 1). Although the seeing was good at CFHT we are faced with a major difficulty when trying to get a point spread function for stars (seeing disk) with small anisotropic deviations from circularity less than $b/a = 0.05$ in every direction). This limitation on the measurement of the weak shear amplitude will be discussed more explicitly in the following section.

3. Measurement of the shear

The measurements of the shear patterns have been obtained from an average of the centered second order momenta as computed by Bonnet and Mellier (1995) of all individual galaxies in a square aperture (scanning aperture size: 57 +3/-5 arcsec.) containing at least 25 faint galaxies with V between 25 and 27.5 (Table 2). Because very elongated objects increase the dispersion of the measurement of the averaged shape parameters (see Bonnet and Mellier 1995, Fig. 4), and blended galaxies give wrong ellipticities, we rejected these objects from the samples. The direction of the polarization of background galaxies is plotted on each QSO field (Figures 3b, 3d, 4b, 5b, 6b) at the barycentre of the 25 background galaxies that are used to calculate the averaged shear. Each plot has the

Fig. 2. Histogram of the independent measurements of the axis ratio b/a in all the fields with a scanning aperture of 30 arcsec. radius. The peak around 0.99 is representative of the noise level that defines a threshold of amplitude detection near 0.985.

Fig. 1. Figure 1a: NTT Field of view of PKS1741 which was used as a star template to study the instrumental distortion of the SUSI camera. Figure 2b: plot of the apparent residual "shear amplitude" of the stars on 5 points of the field where the galaxy shears are determined in other NTT images; figures 4, 5, 6

same amplitude scale for comparison between images and the instrumental distortion found from a star field analysis (Figure 1b). This explains why the mapping is not rigorously made with a regular step between each polarization vector on the figures. The small step variation reflects the inhomogeneity of the distribution of background sources. For the exceptional shear pattern of Q1622+238, a plot with a smaller sampling in boxes of 22 arcsec. gives a good view of the coherence of the shear (Figure 3b). All other maps are given with a one arcminute box, including figure 3d, so that each measurement of the shear is completely independent. For quantitative study the coordinates of each measurement are given on table 3 with the value of the apparent amplitude $1-b/a$ and the direction of the shear. The ellipticity $e = 1 - b/a$ given in Table 3 is drawn on the various fields with the same scale.

A description of the technique used to map the shear can be found in **?**bonnet95. We have only improved when necessary the method to correct the instrumental distortion in order to detect apparent shear on the CCD images down to a level of about 2.0% (Figure 2). Notice that we call here "apparent shear" the observed shear on the image which is not corrected for seeing effects and which is averaged within the scanning aperture. To achieve this goal we observed at NTT, in similar conditions as other radiosources, the field of PKS1741-03 which contains approximately 26 ± 6 stars per square arcminute (Figure 1a,b). After a mapping of the instrumental distortion of stars we have seen that prior to applying the original **?**bonnet95 method, it is possible to restore an ideal circular seeing disk with a gaussian distribution of energy for stars in the field (pseudo deconvolution). The correction almost gives conservation of the seeing effective radius with:

$$s = \sqrt{< a^* > \times < b^* >}\,, \tag{1}$$

where $< a^* >$ and $< b^* >$ are the mean of the major and minor axes of individual stars in the field.

With SUSI the original spread function measured from stars is relatively stable over the 2.2×2.2 square arcmin. field and can be extraordinary good. The blurring of images due to the rotator accuracy when compensating for field rotation was found to be less than 0.004"/hour at the edge of the field, and the tracking errors are often nearly 0.04"/hour.

This level of instrumental distortion corresponds to a mean ellipticity of all stars in the field of $1- < b/a > = 0.015$. Therefore Bonnet & Mellier's method of correction can generally be used directly without significant changes in the results. After the correction, the systematic residual instrumental distortions are often below the intrinsic sensitivity of the method, which is limited by the unknown distribution of source shapes within the resolution aper-

ture (Figure 1b). However we verify with the PKS1741-03 field that the restoration of the circularity of the spread function can give a residual "polarization" of stars in the field as low as $1- < b/a >= 0.0009 \pm 0.0048$ (dispersion).

In fact the restoration of the point spread function appeared to be more difficult with CFHT images because of a higher level of instrumental distortion whose origin is not yet completely determined: guiding errors, atmospheric dispersion, larger mechanical flexure of a non-azimuthal telescope, 3 Hz natural oscillation of the telescope (P. Couturier, private communication), optical caustic of the parabolic mirror, and indeed greater difficulties in getting excellent image quality on a larger field. Thus, the level of instrumental distortion measured on stars is currently $1- < b/a > =$ 0.08-0.12 with complex deviations from a circular shape. After the restoration of an ideal seeing spread function we are able to bring the shear accuracy of CFHT images to a level of 0.03. But like the classical measurement of light polarization it should be far better to start the observations with a level of instrumental polarization as low as possible.

In summary we are now able to reach the intrinsic limitation of Bonnet & Mellier's method on the measurement of the shear amplitude at NTT with a typical resolution of about 60 arcsec. diameter (25-30 faint galaxies per resolution element) with a rms error of about 0.015 (Figure 2). Below this value the determination of the amplitude of the shear is meaningless although the direction may still be valid. At CFHT the detectivity is almost two times less but the field is larger. We are currently developing methods to correct the instrumental distortion at the same level we get with the NTT. This effort is necessary for future programmes with the VLT which would be aimed toward the mapping of Large Scale Structures (shear of 0.01) with a lower spatial resolution (> 10 arcminute apertures).

4. Results

In this section we discuss the significance of the shear pattern in each QSO field and the eventual correlation with the isopleth or isodensity curves of background galaxies with $20 < V < 24.5$. For a fair comparison both the isopleth (surface density numbers) or isoluminosity curves (isopleth weighted by individual luminosity) are smoothed with a gaussian filter having nearly the resolution of the shear map (40" FWHM).

1. *Q1622+238*
   A coherent and nearly elliptical shear pattern is detected with an apparent amplitude 0.025± 0.015 at a distance ranging from 50" to 105" of the QSO (Figure 3b). The center of the shear can be calculated with the centering algorithm described by Bonnet & Mellier. The inner ellipses in figure 3b show the position of the center at the 1, 2 and 3 σ confidence level. It coincides with a cluster of galaxies identified on the deep V image 10 arcsec South-East from the QSO (Figure 3c). The external contour of the isopleth map in figures 3c corresponds to a density excess of galaxies of twice the averaged values on the field for a 30 arcsec circular aperture. The isoluminosity map shows a light concentration even more compact than the number density map. About 70% of the galaxies of the condensation have a narrow magnitude range between $V = 24$ and 24.5 and are concentrated around a bright galaxy with $V = 21.22 \pm 0.02$. This is typical for a cluster of galaxies. A short exposure in the I band gives a corresponding magnitude $I = 19.3 \pm 0.1$ for the bright central galaxy. A simple use of the magnitude-redshift relationship from a Hubble diagramme and the $(V - I)$ colors of the galaxy suggest a redshift larger than 0.5. By assuming such a redshift

Object	field (pixels)/(arcsec.)	Ng/N$_G$	Mag range
PKS0135-247	930×970 / 121×126	112/48	22-25
PKS1508-05	951×947/124×123	152/33	22-24.5
PKS1741-03	888×818/115×106	99/27	22-24.5
3C446	925×946/120×123	102/65	22-25
Q1622+238	1871×1855/385×382	1652/486	21-24.5

Table 2. Table 2: Number Ng of (background) galaxies from V=22 to 24.5 which are used to trace isopleth and number N$_G$ of (distant) galaxies from V=25 to 27.5. detected on each observed field

   it is possible to mimic the shear map with a deflector velocity dispersion of at least 500 km/sec. After a correction for the seeing effect with the **?**bonnet95 diagram and taking into account the local shear of the lens at the exact location of the QSO we can estimate that the magnification bias could be exceptionally high in this case (> 0.75 magnitude). Further spectrophotometric observations of the field are needed to get a better description of the lens. It is even possible that multiply imaged galaxies are present at the center of this newly discovered cluster.

2. *PKS1741-03*
   This first NTT field was chosen for a dedicated study of the instrumental distortion of the SUSI instrument. Indeed it is crowded with stars and the mapping of the isopleth was not done due to large areas of the sky occulted by bright stars.
   The center of the field of PKS1741-03 shows a faint compact group of galaxies (marked g on fig 1a). A detailed investigation of the alignment of individual faint galaxies nearby shows that a few have almost orthoradial orientation to the center of the group. The amplitude of the "apparent" shear on the fig 1b is low probably because it rotates within the scanning aperture around a deflector having an equivalent velocity dis-

Fig. 3. Figure 3a: CFHT field of view of Q1622+238 in V. North is at the top. Figure 3b: Shear map of Q1622+238 with a resolution step of 22 arcsec. The ellipses shows the position of the center of the central shear with the 1, 2, 3 σ confidence level. The center almost coincides with a distant cluster clearly visible on figure 3c.

persion lower than 400 km/s. Outside the box the apparent shear is already below the $1- < b/a >= 0.015$ threshold level and it is not possible to detect the circular shear at distance from the group larger than one arcmin. This remark is important because it illustrates the limitation of the method in detecting lenses with a $1- < b/a >= 0.015$ on angular scales smaller than the scanning aperture. Therefore a low amplitude of the shear on the scanning aperture could be the actual signature of a small deflector rather than a sky area with a low shear! Although the compact group is only 30 arcsec South-East of the QSO it might contribute to a weak lensing of PKS1741-03 but it is difficult to get a rough estimate of the amplitude of the magnification bias.

3. *PKS1508-05*
This is the second bright radiosource of the sample. At one arcminute North-West there is also a group around a bright galaxy (G) which could be responsible for a large shear. This distant group or cluster may contribute to a weak magnification by itself, but there is also a small clump of galaxies in the close vicinity of the radiosource with the brightest member at a distance of 8 arcseconds only. The situation is similar to the case of the multiple QSO 2345+007 (BFK$^+$93). This could be the dominant lensing agent which provides a larger magnification bias, especially if the nearby cluster has already provided a substantial part of the critical projected mass density.

4. *3C446*
The radiosource is among the faintest in the optical (table 1). There is a loose group of galaxies at 40 arcsec South-West from the QSO. The orientation of the shear with respect to the group of galaxies can be reproduced with a rough 2D simulation (Hue95) although at first look it was not so convincing as the PKS0135-247 case. The lensing configuration could be similar to PKS1508-05 with a secondary lensing agent G near the QSO (fig 6a,b). Surprisingly there is also a large shear amplitude which is not apparently linked to an overdensity of galaxies in V in the North-East corner. In such a case it is important to confirm the result with an I image to detect possible distant groups at a redshift between 0.5 and 0.7. *A contrario* it is important to mention that the shear is almost null in the North-West area of the field which actually has no galaxy excess visible in V (fig 6b).

Fig. 4. Figure 3c: Zoom at the center of the field of view of Q1622+238. The distant cluster around the bright central elliptical galaxy E is clearly identified on this very deep V image. Figure 3d: Shear map of Q1622+238 with a resolution step of 60 arcsec. similar to the resolution on other NTT fields. The ellipses shows the position of the center of the central shear with the 1, 2, 3 σ confidence level. The center almost coincides with a distant cluster clearly visible on figure 3c.

5. Discussion

Due to observational limitations on the visibility of radiosources during the observations the selection criteria were actually very loose as compared with what we have proposed in Section 2 for a large survey. The results we present here must be considered as a sub-sample of QSOs with a moderate possible bias. Nevertheless, for at least 3 of the sources there are some lensing agents which are associated with foreground groups or clusters of galaxies that are detected and correlated with the shear field. For the 2 other cases the signature of a lensing effect is not clear but cannot be discarded from the measurements. All the radiosources may have a magnification bias enhanced by a smaller clump on the line of sight or even an (unseen) foreground galaxy lying a few arcsec from the radiosource (compound lens similar to PKS1508). The occurrence of coherent shear associated with groups in the field of the radiosources is surprisingly high. This might mean that a lot of groups or poor clusters which are not yet identified contain a substantial part of the hidden mass of LSS of the Universe below $z = 0.8$. Some of them responsible for the observed apparent shear may be the most massive progenitor clumps of rich clusters still undergoing merging.

Although these qualitative results already represent a fair amount of observing time we are now quite convinced that all of these fields should be reobserved, in particular in the I and K bands, to assess the nature of the deflectors. Spectroscopic observation of the brightest members of each clump is also necessary to determine the redshift of the putative deflectors. This is an indispensable step to connect the shear pattern to a quantitative amount of lensing mass and to link the polarization map with some dynamical parameters of visible matter, such as the velocity dispersion for each deflector, or possibly the X-ray emissivity. at the present time, we are only able to say that there is a tendency for a correlation between the shear and light overdensity (FM94).

From the modelling point of view, simulations have been done and reproduce fairly well the direction of the shear pattern with a distribution of mass that follows most of the light distribution given by the isopleth or isoluminosity contour of the groups in the fields. Some of these condensations do not play any role at all and are probably too distant to deflect the light beams. Unfortunately, in order to make accurate modelling it is necessary to have a good estimate of the seeing effect on the amplitude of the shear by comparing with HST reference fields, and good redshift determinations as well of the possible lenses to get their gravitational weight in the field. It is also important to consider more carefully the effect of convolution

Fig. 5. Figure 4a: NTT field of view for PK0135. North is at the top. Note the group of galaxies around g1, g2, g3 and g4 responsible for a coherent shear visible on figure 4b

Fig. 6. Figure 5a: NTT field of view for PKS1508. Note the North-West group of galaxies near the brighter elliptical E responsible for a larger amplitude of the shear on figure 5b and the small clump of galaxies g right on the line of sight of the QSO.

of the actual local shear which varies at smaller scales than the size of the scanning beam (presently about one arcminute size). This work is now being done but is also waiting for more observational data to actually start to study the gravitational mass distribution of groups and poor clusters of galaxies in the field of radiosources.

6. Conclusion

The shear patterns observed in the fields of five bright QSOs, and the previous detection of a cluster shear in Q2345+007 (BFK$^+$93) provide strong arguments in favor of the **?**bartelmann93b hypothesis to explain the large scale correlation between radiosources and foreground galaxies. The LSS could be strongly structured by numerous condensations of masses associated with groups of galaxies. These groups produce significant weak lensing effects that can be detected. A rough estimate of the magnification bias is given by the polarization maps around these radiosources. It could sometimes be higher than half a magnitude and even much more with the help of an individual galaxy deflector at a few arcsec. of the QSO line of sight. The results we report here also show that we can study with the weak shear analysis the distribution of density peaks of (dark) massive gravitational structures (ie $\sigma > 500$ km/s) and characterise their association with overdensities of galaxies at moderate redshift (z from 0.2 to 0.7).

A complete survey of a large sample of radiosource fields will have strong cosmological interest for the two aspects we mentioned above. Furthermore, the method can be used to probe the intervening masses which are associated with the absorption lines in QSOs or to explain the unusually high luminosity of distant sources like the ultraluminous sources IR 10214+24526 (SLR$^+$95) or the most distant radio-galaxy 8C1435+635 (z=4.25; LMR$^+$94).

Therefore we plead for the continuation of systematic measurements of the shear around a sample of bright radiosources randomly selected with the double magnification bias procedure (BvR91). Our very first attempt encountered some unexpected obstacles related to the limited field of view of CCDs or the correction of instrumental distortion. It seems that they can be overcome in the near future. We have good hopes that smooth distributions of mass associated with larger scale structures like

Fig. 7. Figures 6a: NTT field of 3C446. Note on figure 6b the shear pattern relatively to the isopleth of possible foreground groups and the galaxies g on the line of sight of the QSO

filaments and wall structures could be observed with a dedicated wide field instrument that minimizes all instrumental and observational systematics, or still better with a Lunar Transit telescope (FMV95).

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x	y	angle (degrees)	1-b/a
PKS0135-247			
274	202	-83	0.012
258	741	0	0.027
698	228	67	0.006
713	681	42	0.026
473	408	69	0.021
PKS1508-05			
270	272	73	0.012
216	712	-62	0.009
704	251	-65	0.026
745	695	-51	0.032
453	463	-85	0.016
PKS1741-03			
255	268	12	0.013
256	613	-56	0.013
694	286	-63	0.006
716	620	-80	0.017
473	425	37	0.003
3C446			
247	261	86	0.014
284	707	74	0.027
688	271	66	0.015
720	693	70	0.002
452	421	-69	0.021
Q1622+238			
209	187	-54	0.005
201	583	16	0.006
203	978	-50	0.008
196	1399	-28	0.013
259	1665	-46	0.009
629	201	-50	0.019
588	596	-65	0.033
610	996	53	0.014
603	1398	4	0.022
616	1682	4	0.006
984	194	24	0.013
999	590	14	0.015
996	979	65	0.010
992	1402	20	0.013
983	1718	-20	0.025
1407	202	-43	0.013
1386	598	78	0.051
1403	999	13	0.002
1407	1401	42	0.013
1420	1708	-69	0.012
1698	210	52	0.015
1715	595	44	0.022
1723	1027	33	0.025
1716	1387	41	0.014
1650	1676	-49	0.006

Table 3. Summary of relevant data for weak lensing measurements. X and y are the averaged center positions (in pixel) of each independent box shown in the previous figures. The orientations and the axis ratios also correspond to the values associated with each individual line of the figures.

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